CONSTRUCTING GEOMETRIC PROPERTIES OF RECTANGLE, SQUARE, AND TRIANGLE IN THE THIRD GRADE OF INDONESIAN PRIMARY SCHOOLS

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Abstract

Previous studies have provided that when learning shapes for the first time, young children tend to use the prototype as the reference point for comparisons, but often fail when doing so since they do not yet think about the defining attributes or the geometric properties of the shapes. Most of the time, elementary students learn geometric properties of shapes only as empty verbal statements to be memorized, without any chance to experience the concepts meaningfully. In the light of it, a sequence of instructional activities along with computer manipulative was designed to support Indonesian third graders in constructing geometric properties of square, rectangle, and triangle. The aim of the present study is to develop a local instructional theory to support third graders in constructing geometric properties of rectangle, square and triangle. Thirty seven students of one third grade classes in SD Pupuk Sriwijaya Palembang, along with their class teacher, were involved in the study. Our findings suggest that the combination of computer and non computer activities supports third graders in constructing geometric properties of square, rectangle, and triangle in that it provides opportunities to the students to experience and to develop the concepts meaningfully while using their real experiences as the bases to attain a higher geometric thinking level.

Key concepts: Geometric properties, rectangle, square, triangle, design research, realistic mathematics education.

Abstrak

Banyak penelitian yang telah dilakukan menyatakan bahwa ketika siswa belajar bangun datar untuk pertama kalinya, siswa cenderung menggunakan purwarupa sebagai acuan dalam membandingkan sifat-sifat bangun datar, namun seringnya siswa gagal ketika melakukannya karena mereka belum berfikir tentang sifat-sifat geometri bangun datar. Seringnya, siswa Sekolah Dasar (SD) belajar sifat-sifat geometri bangun datar hanya sebagai pernyataan verbal kosong yang harus dihafalkan, tanpa ada kesempatan untuk mengalami konsep tersebut secara bermakna. Oleh sebab itu, serangkaian kegiatan pembelajaran beserta dengan alat peraga komputer dirancang untuk mendukung siswa kelas 3 SD di Indonesia dalam mengkonstruksi sifat-sifar geometri dari persegi, persegi panjang, dan segitiga. Tujuan dari penelitian ini adalah untuk mengembangkan teori pembelajaran lokal untuk mendukung siswa kelas 3 SD dalam mengkonstruksi sifat-sifar geometri dari persegi, persegi panjang, dan segitiga. Tiga puluh tujuh siswa kelas 3 SD Pupuk Sriwijaya Palembang dan guru kelas mereka, terlibat dalam penelitian ini. Temuan kami menyarankan bahwa kombinasi dari aktivitas komputer dan non komputer mendukung siswa kelas 3 SD dalam mengkonstruksi sifat-sifar geometri dari persegi, persegi panjang, dan segitiga, dengan cara menyediakan kesempatan bagi siswa untuk mengalami dan mengembangkan konsep-konsep tersebut secara bermakna pada saat mereka menggunakan pengalaman yang dimiliki sebagai dasar untuk mencapai level berfikir geometri yang lebih tinggi.

Kata kunci: Sifat-sifat geometri, persegi panjang, persegi, segitiga, design research, realistic mathematics education

Shape is a fundamental concept in cognitive development (Clements & Sarama, 2009). Shape is also a fundamental idea in geometry and other areas of mathematics. Some previous studies (Fox, 2000; Hannibal, 1999; Schifter, 1999; Clements et al., 1999) have provided that when learning shapes for the first time, young children tend to develop their own prototype for simple shapes such as triangle, square, rectangle, and circle. Children use the prototypes as the reference points for comparisons (Hannibal, 1999). In making comparisons, children often fail if the shapes are in different orientation, ratio, skewness or size (Aslan & Arnas, 2007). They do not yet think about the defining attributes or the geometric properties of the shapes. Most of the studies on children and shapes such as Clements et al. (1999) and Aslan & Arnas (2007) deal with children age 3-6 years old. There is no study about how third graders, age 8-9, construct geometric properties of simple geometric figures, such as square, rectangle, and triangle, that they can use to develop their geometric thinking (Fuys et al., 1984) from level 0 (visualization) to level 1 (analysis) and later level 2 (abstraction). Studies in this area are really needed since in Indonesia, third grade is the first time for the students to deal with this kind of concept and it is a good opportunity to encourage the students to think mathematically and logically. What is known from Battista (2001), elementary students learn geometric properties of shapes as empty verbal statements to be memorized, without any chance to experience it meaningfully.

Based on the idea of logo turtle of Pappert (1980), in which its combination with non computer activities designed to help students abstract the notion of path already put into practice by Clements et al. (1997) and empirically succeeds in providing a fertile environment for developing students' geometric thinking of simple two dimensional figures, a sequence of activities along with a computer manipulative will be designed to support Indonesian third graders in constructing geometric properties of square, rectangle, and triangle. In this study, it will be investigated how a sequence of instructional activities along with a computer manipulative designed for grade three Indonesian students can support them in constructing geometric properties of rectangle, square and triangle. The aim of this study is to develop a local instructional theory to support third graders in constructing geometric properties of rectangle, square and triangle with the research question: *How can a combination of computer and non computer activities support third graders in constructing geometric properties of square, rectangle, and triangle?*

When learning shapes for the first time, young children tend to develop their own prototype for simple shapes, such as triangle, square, rectangle, and circle (Fox, 2000; Hannibal, 1999; Schifter, 1999; Clements et al., 1999). For those shapes, the prototype is the "real" or "perfect" figure (Hannibal, 1999). Students use the prototype as the reference point for comparisons (Hannibal, 1999). In making comparisons, children often fail if the shapes are in different orientation, ratio, skewness or size (Aslan & Arnas, 2007).

Geometric properties define a relationship between parts of a shape. These geometric properties are established by observing, measuring, drawing, and model making (Clements & Sarama, 2009). All these four aspects are used as the bases on designing instructional activities and computer manipulative in the present study. The geometric properties can be used to develop students' geometric thinking in that it supports students in understanding the classes of figures. Understanding the classes of figures can help students to not fall into common misconception that a square is not a rectangle (Battista, 2001; Erez & Yerushalmy, 2006; Clements & Sarama, 2009). The van Hiele theory of geometric thinking consists of five levels of understanding, numbered 0 through 4. In table 1, adapted from Mistretta (2000), it will be given the 5 levels and characteristics of each.

Table 1. Characteristics of van Hiele levels of geometry thinking

Level	Characteristics
Level 0 Visualization	Students recognize figures by appearance alone, seen as a total entity, often by comparing them to a known prototype.
Level 1 Descriptive/ Analysis	Students identify figures by their geometry properties, rather than by its appearance. They can recognize and name properties of geometric figures, but they do not see relationships between these properties.
Level 2 Abstraction/ Informal deduction	Students perceive relationships between properties and between figures. At this level, students can create meaningful definitions and give informal arguments to justify their reasoning.
Level 3 Formal deduction	Logical reasoning ability is developed. Geometric proofs are constructed with meaning. Necessary and sufficient conditions are utilized with strong conceptual understanding.
Level 4 Rigor	Students can compare axiomatic systems. Theorems in different postulation system are established and analyzed

In the relation with logo activities, Clements & Sarama (1995) found that logo activities might be used to encourage students to progress from visual level to descriptive/analysis level. Using logo like manipulative designed, students will be asked to construct a sequence of commands to draw a rectangle. This activity allows or obliges the students to do what Papert (1980) calls externalize intuitive expectation.

Mathematics has changed universally. There is shift from teaching as transmission of knowledge toward learning as construction of knowledge (Gravemeijer, 2010). Freudenthal (1991) also gives an emphasis on the construction of knowledge. He asserted that mathematics must be viewed as 'a human activity' instead of a ready-made product. That is why mathematics should be taught in such a way, so students by themselves can do and experience mathematics to grasp the concepts. In the light of it, this study designs a sequence of instructional activities and a computer manipulative, namely "Jejak Si Kura" (The turtle‟s path). This computer manipulative serves as an aid on teaching and learning two dimensional figures in which the students could gain more insight about paths, shapes and their geometrical properties through experiencing a sequence of meaningful activities instead of only memorizing empty verbal statements written on the text books or given by the teacher. In designing the computer manipulative and sequence of following instructional activities, it is consulted to five tenets of realistic mathematics education (de Lange, 1987; Gravemijer, 1994), which are described as following:

a. Phenomenological exploration. A contextual situation, parade activity, serves as the preliminary activity.
b. Bridging by vertical instrument. In the present study, the development from concrete to more abstract level can be seen as the shift in students' reasoning related to their geometric thinking level.
c. Students' own constructions and productions. Students are free to discuss what strategies they are going to use and why a certain shape can or cannot be formed by a sequence of orders.
d. Interactivity. Students will experience the activities and discuss together to come to the geometric properties of rectangle, square, and triangle.
e. Intertwinement. The computer manipulative and instructional activities designed not only support learning for simple geometric figures: paths, rectangles, squares, triangles, and the geometry properties of certain shapes, but also support geometric processes: measuring, turning, visualizing and arithmetic: computation and estimation.

A computer manipulative lesson requires the same amount and quality of planning as any others mathematics lessons (Bell, 1978), otherwise we will only modernize the lesson without making any improvement in learning (Batista, 2001). That is why it needs a sequence of instructional activities that makes sure it can be used properly in order to foster students' understanding in mathematics concepts. Papert (1980) discusses turtle geometry as ego-syntonic or fitting the ways of thinking of the child as a geometric knowledge builder, while Clements et al. (1997) provide evidence that a combination of turtle based manipulative and non computer activities could provide a fertile environment to develop students' geometric thinking of two dimensional figures.

The designed computer manipulative will also support students' development and use of appropriate mental models for dealing with physical, conceptual, and symbolic mathematical phenomena as suggested by Battista (2001). Since mathematical conceptualizations and associated mental models result from reflection on and abstraction of student's own mental actions, computer manipulative must make those actions and their consequences more accessible to reflection. The gap between student's predictions and what actually happens provide a constant source of perturbations requiring accommodations that lead to increasingly sophisticated conceptions (Battista, 2001).

The geometry lesson concerning on shape is already given from the first grade in Indonesia, but it is not until in the third grade the students deal with the geometric properties of the shapes. The geometry and measurement curriculum for the third grade in Indonesia which is developed by National Bureau for Educational Standardization (BNSP) is given in Table 2.

Table 2. Geometry and measurement curriculum for third grade in Indonesia

Competency standard	Basic competencies
Geometry and Measurement 4. Understanding elements and properties of simple two-dimensional figures	4.1 Identifying various simple two-dimensional figures based on their properties and elements 4.2 Identifying various angles based on their types and magnitudes

METHOD

Research approach

Considering the aim of the present study, design research is a research approach which is suitable to answer the research question and to contribute to the research aim. Gravemeijer & Cobb (2006) define design research by discussing the three phases of conducting a design experiment which are preparing the experiment, experimenting in the classroom, and retrospective analysis.

Data collection

1. Preparation phase. There were some types of data collected in this phase, namely classroom observation, study material, pre-test, interview with students and teacher.
2. Preliminary teaching experiment. Data collection in this phase was aiming at trying out the initial HLT designed. For these purposes, data from classroom observation and students' work were collected.
3. Teaching experiment. Data collection in this phase was aiming at trying out the HLT that was already improved based on the result of the preliminary teaching experiment. Some types of data which were collected consisted of pre-test, classroom observation, group observation, students' work.
4. Post-test. The post-test administered was aiming at getting information about what the students had learned during the preliminary teaching experiment or the real teaching experiment. The written post-test, given to all students, later on was followed by an interview related to their answers and thinking processes.

Data Analysis

1. Pre-test. The written pre-test answers and strategies from the students were examined. These starting points were used to adjust the HLT already designed so that the learning activities were suitable to the students' levels of understanding.
2. Preliminary teaching experiment. Data of classroom observation and the students' works gathered in this phase were analyzed. The purpose of the analysis was to know to what extent the actual learning taking place matches with what was conjectured in the HLT and to found new insights that were used to refine the HLT and to improve the practice in the next cycle.

3. Teaching experiment. Data of classroom observation, group observation and the students' works collected in this phase were analyzed. The analysis was done in order to get information that was used to answer the research question, to draw a conclusion, and to redesign the HLT.
4. Post-test. The post-test was analyzed by examining the result of students' written post-test and the interview results related to the post-test. The result of the analyses contributed to the conclusions of the present study.

Research Subject

The present study was conducted in class 3B SD Pusri Palembang with 37 students in it, including the 4 focus group students, and Bu Melda as the class teacher. In total there were 8 meetings: 6 for the lessons, 1 for pre-test, and 1 for post-test. There was also a pilot study conducted before it. The pilot study involved 4 students from class 3C SD Pusri Palembang and also had 8 meetings in total.

Retrospective Analysis

Preliminary Teaching Experiment

The pre-test given to the four students involved in the first cycle exactly gave result matching what is already conjectured before. The students based their categorization of shapes on the shapes' overall appearance. No attention was paid to the shapes geometrical properties. They could not explain why a certain shape is a rectangle or a square. They could categorize squares and rectangles with a horizontal base, but when it was slightly rotated, they did not see it as a square or a rectangle anymore. Similar thing happened to the categorization of the triangle task. Related to the van Hiele level of geometrical thinking, it could be inferred that these students were in the level 0, visualization.

In the parade activity, students were doing activity in the school yard. In a group of four, one student became the commander, another became the soldier, and the other two became the secretary and the drawer. As conjectured, by taking some time in doing the activity, the students could realize that they could make rectangle and square. They also realized that shape consists of parts, sides and vertices. In addition they could argue that the sides of rectangle or square have to be straight lines.

After doing parade activity, the students were asked to do almost the same activity, but on the grid paper. The students were helped by the grid to structure square and rectangle they made. They also could perceive the length structure of rectangle and square as conjectured and got insight that the opposite sides have to be in the same length.

In triangle activity, students started working with the computer manipulative the turtle path. This manipulative has similar idea with the parade activity. The differences are the soldier becomes a turtle and right face command is replaced with turn command which has two directions, left and right, and also magnitudes. By using this manipulative the students have opportunities to create more shape such as an equilateral triangle.

Students could define angle using their own word. For example one student said that angle is two lines that meet in one point. They could also define right angle and give example how to determine whether an angle is a right one by using one corner of the paper. Related to the geometric properties of triangle, the students could say that a shape is a triangle if it has three sides and three vertices as already conjectured before.

The students also worked with the second manipulative, namely triangles drag. The students chose any three points on the screen. These three points will be connected to one another to form a triangle. After that, the students were asked to drag the triangle's vertices in order to form an interesting figure. They had to determine whether the new figure they made is a triangle or not. Since the students never dealt with non exemplars triangles, they said that some figures they made not triangles. Their reasons are the shape is too pointy, it is very thin, and it does not look like a triangle. All this reactions are already conjectured before. After having discussion about what is needed by a figure in order to be a triangle, they could come to the conclusion that as long as one shape has three sides and three vertices, it is a triangle.

Working with the computer manipulative in making rectangles and squares, students found the regularities of those two and investigated their relation based on their geometric properties. Students gave respond as conjectured in the Hypothetical Learning Trajectory (HLT); they could conclude that a square is also a rectangle.

In the following activity, students were challenged to write down as many as possible commands needed to form a rectangle whose total length of it sides is 200. As they did in the previous activity, they started to do trial and error. They found problem since the number is a big one for them. After the number was changed to 20, they could do the task well. Starting with trial and error, they could utilize the geometric properties of rectangle and square to find many possible commands, but they could not yet conclude that for rectangle the sum of adjacent sides is always one half of the total length of all sides.

The result of the post-test in the pilot study informed that 3 out of 4 students already grasped the geometric properties of rectangle, square, and triangle. They could determine whether one shape is a triangle, a rectangle, or a square base on its geometric properties. They could also give reasoning why a certain shape, for example, is a rectangle by mentioning its geometric properties. They could also solve problems related to the shapes' geometric properties. Compared to the result of the pre-test, it could be said that they already construct the geometric properties of rectangle, square, and triangle and already in a higher level of geometrical thinking, in this case level 1 analysis.

The Improved HLT

Based on findings in the teaching experiment phase, some activities are revised in order to get optimal results. For the parade activity, the students will be given symbols to record the commands they use since it costs a lot of time to write down the commands completely. From the beginning of the activity, the students will be suggested to use M, stands for *maju* (Indonesian word for forward), to record go forward commands and HK, stands for *hadap kanan* (right face), to record right face command. Another improvement for the first activity is the soldier students should give mark to the point where they stand, using small paper, before doing a right face command, so the students could explicitly see vertices of the shapes they make and will not loss track.

For paper parade activity, improvement is addressed to the size of the square on the grid paper. One reason to make the square on the grid paper bigger is the expectation that the students will be really helped to structure shapes they make and perceive the length structure of rectangle and square. In working with triangle activity, the improvement is to make 100 pixels equal to 1 step.

In finding the relation between rectangle and square, the improvement is addressed to the students' worksheet. A two-column table is provided. The first column is used to list all the regularities of rectangle resulted from the students' observation while the other one is used for those of square, so the students can easily compare and establish a relation between those two shapes. For the last activity, paths with the same length, the improvement is reducing the length from 200 to 20, so the students can more focus on the geometric properties of the rectangle.

Teaching Experiment

The result of pre-test given to all 37 students participating the teaching experiment phase informed more or less the same thing as what was acquired from the pre-test for the four students in the preliminary teaching phase. All the students failed to identify rotated squares as squares. It also happened for rectangle and triangle. They could not make good categorization of shapes. They based their categorization solely on the shapes' whole appearances. Hence, according to van Hiele level of geometrical thinking, it could be said that they were still on the level 0, visualization. Asked whether a square is also a rectangle, all the students said no and most of them gave reason that indicated they were on level 0, reasoning. No attention was paid to the shapes' geometrical properties.

In the parade activity, as conjectured, the students were able to form rectangle and square. They knew that each command will affect the resulted figure and the same commands will also yield the same figures. They knew that with the right face command if all the sides are in the same length, they would have a square. In the next activity, students working in pair were supposed to give commands as they did in the first activity, but their soldier now was their pencil that walks on the grid paper

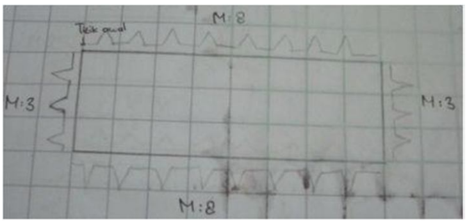

Figure 1. Student's drawing

The use of the grid paper really helps the students to structure figures they make, as seen from Figure 4.1, so they can recognize the length structure of the shapes and clearly see whether shapes they make rectangle or square. The pattern on the grid also helps the students to recognize the regularity of rectangle's and square's corners. Even though the students have not known the term "right angle" or "angle", they will perceive that the corners of rectangle and square have a special form.

Using computer manipulative, the turtle path, the students were asked to form an equilateral triangle. The students in groups of four were asked to make nine variants of the shapes using this manipulative. Then they have to decide which shapes are triangles. They failed to identify non exemplar triangles such as triangle with non horizontal base or a thin triangle as a triangle. Reflecting to the HLT already conjectured for this activity, almost all the students at the end of the activity succeeded in determining that all the figures are triangle. They also could make a conclusion that a shape is a triangle as long as it has three sides and three vertices.

In the next activity, some groups found some difficulties in the checking task when they investigated the relation between rectangle and triangle. They did not rely only on the geometric properties of the shapes they already established but on their pre-knowledge of rectangle and square. Rectangle has to be long and square has to look like a box. Here will be given one students' answer for checking task, whether a rectangle is a square or not.

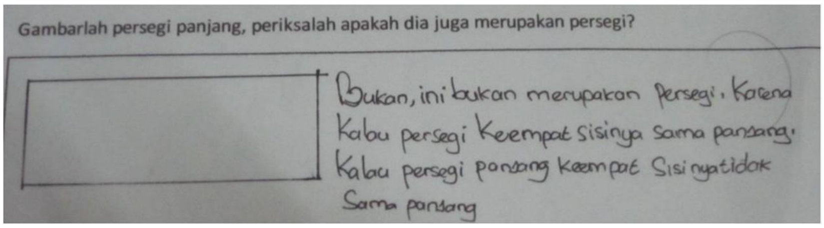

Figure 2. Example of students' answer

Encouraged to make use the geometric properties of rectangle and square, this group was succeeded in determining relations between rectangle and square. They started to get used to use shapes' geometric properties to categorize shape. As conjectured, in making list of properties of the shapes, they listed all that they could see, not only the defining properties. Hence it can be said that they were already in geometrical reasoning level 1, descriptive/analysis. Considering all the students on the class in the end of lesson, after having discussion led by the teacher, all the students could attain that a square is also a rectangle.

In the last activity, the students had to make several different rectangles using the commands that have the same total length of sides, 20. The students also had to make a conclusion related to the sum of the lengths of two adjacent sides of the rectangle and use it to solve 3 problems given. In doing so, all the students did trial and error at the beginning. For example, students from the focus group at the first chose number 7 as the length of the first side. Using the geometrical properties of rectangle that opposite sides have to be in the same length, they added up 7 by 7 to get 14. Since the total is 20, they subtracted 14 from 20 to get 6 and divided it by to 2 to get 3 as the length of the other two sides.

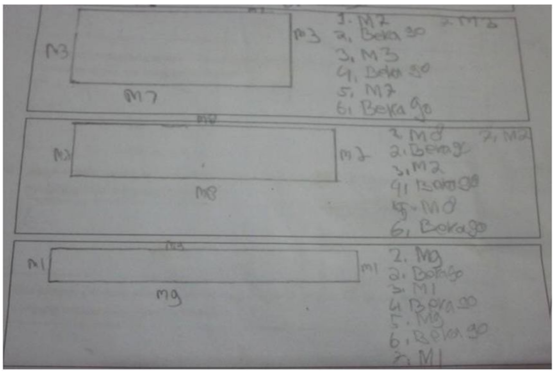

Figure 3. Three rectangles made by the focus group students

These students had correctly applied the geometric properties of rectangle, the equality of the opposite sides, to solve the problem but they had not attained another geometrical property namely the sum of two adjacent sides is equal to one half of the total length of the all sides.

The result of the post-test in the teaching experiment gave insight that some students already grasped the geometric properties of rectangle, square, and triangle. They could determine whether one shape is a triangle, a rectangle, or a square base on its geometric properties. They could also give reasoning why a certain shape, for example, is a rectangle by mentioning its geometric properties not solely by its whole appearance anymore. The relation between rectangle and square were also established. They could also solve problems related to the shapes' geometric properties. Compared to the result of the pre-test, it could be said that they already constructed the geometric properties of rectangle, square, and triangle and already in a higher level of geometrical thinking, in this case level 1 analysis.

CONCLUSION

The parade activity supports the students in that it provides opportunities to the students to reflect shapes they already make using the commands needed to construct those shapes. They could perceive the reasons why the opposite sides of rectangle and square have to be in the same length. On the paper parade activity, which is the replication of the first one but on a paper grid, helps the students to structure rectangle and squares they construct in drawings. The grids on the paper also encourage them to grasp the length structure of the shapes in that two opposite sides of rectangle or square are always in the same length. The structure of the shapes provided by the grids paper that is already grasped by the students could lead them to one conclusion that shape is invariant under rotation.

The first computer manipulative, the turtle path, provides opportunities to the students to construct more shapes than they could do using the commands in the parade activity since they could input numbers indicating the degree of the turn the turtle do. The activity following the use of this manipulative provides the students opportunities to perceive that there is a magnitude determining the relation between two segments having one end in common and define angle for them. The second computer manipulative, the triangle drag, really supports the students in constructing the properties of triangles in that it gives chances to the students to construct more variants of triangles and reflect whether they could identify those triangles as triangle or not.

The fourth activity that asks the students to establish the relation between rectangle and square encourages the students to construct the geometric properties of those shapes. The properties of the rectangle and square already established and recorded in a table will be used to check whether a square is a rectangle and vice versa. Doing this activity, the students will draw a conclusion based on the two shapes geometrical properties that a square is also a rectangle. Succeeding in attaining the geometrical properties of rectangle and square and also the conclusion that a square is a rectangle could develop students' geometrical thinking into a higher level than visualization.

The last activity, paths with the same lengths, gives the students problems that encourage students to construct another geometric property of rectangle, namely the sum of two adjacent sides is one half of the total length of all the sides. This property could lead the students to the reinvention of the formula to find perimeter of one rectangle.

REFERENCES

Aslan, D. & Arnas, Y. A. (2007). Three- to six- year-old children's recognition of geometric shapes. International Journal of Early Years Education, 15(1), 83-104.

Battista, M. T. (2001). Shape makers. Computers in the Schools, 17(1-2), 105-120.

Bell, F.H. (1978). Teaching and learning mathematics in secondary school. Dubuqe, Iowa :Wm C. Brown company.

Chang, K., Sung, Y., & Lin, S. (2007). Developing geometry thinking through multimedia learning activities. Computers in Human Behaviour, 23, 2212-2229

Clements, D. H. & Sarama, J. (1995). Design of a logo environment for elementary geometry. Journal of Mathematical Behaviour, 14, 381-398.

Clements, D. H. & Sarama, J. (2009). Learning and teaching early math: The learning trajectory approach. New York: Rouletge

Clements, D. H., Battista, M. T., Sarama, J., Swaminathan, S., & McMillen, S. (1997). Students" development of length concept in a logo based unit on geometric paths. Journal for Research in Mathematics Education, 28(1), 70-95.

Clements, D. H., Swaminathan, S., Hannibal, M. A. Z., & Sarama, J. (1999). Young children"s concepts of shape. Journal for Research in Mathematics Education, 30(2), 192-212.

Crowley, M.L. (1987). The van Hiele model of the development of geometric thought. In M.M. Lindquist, Ed., Learning and teaching geometry, K-12 (pp. 1-16). Reston, VA: NCTM

de Lange, J. (1987). Mathematics, insight and meaning. Utrecht: OW & OC.

Erez, M. M. & Yerushalmy, M. (2006). "If you can turn a rectangle into a square, you can turn a square into a rectangle …" young students experience the dragging tool. International Journal of Computers for Mathematical Learning, 11, 271-299.

Fox, T.B. (2000). Implications of research on children"s understanding of geometry. Teaching Children Mathematics, 6, 572-576.

Freudenthal, H. (1991). Revisiting Mathematics Education: China Lectures. Dordrecht: Kluwer Academic Publishers.

Fuys, D., Geddes, D., & Tischler, R. (Eds.). (1984). English Translation of Selected Writings of Dina van Hiele-Geldof and Pierre M. Van Hiele. Brooklyn: Brooklyn College.

Gravemeijer, K. (1994). Developing Realistic Mathematics Education. Utrecht: CD Beta Press.

Gravemeijer, K., & Cobb, P. (2006). Design research from the learning design perspective. Educational design research (pp. 17-51). London: Routledge.

Gravemeijer, K.P.E (2010). Realistic Mathematics Education Theory as a Guideline for Problem-Centered, Interactive Mathematics Education. In Sembiring, R. K., Hoogland, K., & Dolk, M., (Eds), A Decade of PMRI in Indonesia, Bandung, Utrecht: APS International.

Gurevich, I., Gorev, D., & Barabash, M. (2005). The computer as an aid in the development of geometrical profiency: a different approach. International Journal of mathematics education in Science and Technology, 36(2-3), 286-302

Hannibal, M.A. (1999). Young children"s developing understanding of geometric shapes. Teaching Children Mathematics, 5, 353-357.

Mistretta, R. M. (2000). Enhancing geometric reasoning. Adolescence, 35(138), 365-379.

Papert, S. (1980). Mindstorms: Children, computers and powerful ideas. Brighton, Sussex : Harvester Press.

Schifter, D. (1999). Learning geometry: Some insights drawn from teacher writing. Teaching Children Mathematics, 5, 360-366.